Physics 208 Exam 2 **Name** Solution

You are graded on your work, with partial credit. (Be sure to include the proper units in each answer.)
The answer by itself is not enough, and you receive credit only for your work.
Please be clear and well-organized, so that we can easily follow each step of your work.
See the last page of the exam for the formula sheets.

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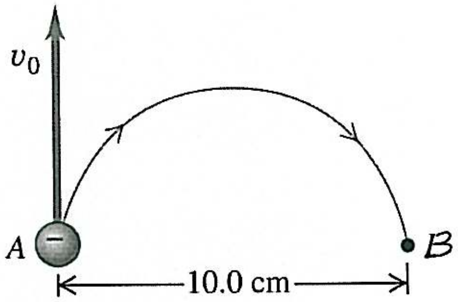
1. An electron at point A in the figure has speed v_0. The magnetic field has an unknown direction and unknown magnitude B. The mass and charge of the electron are m and $-e$. The radius of the semicircular path of the electron is $R = 5.0$ cm.



(a) (2) What is the magnitude of the force F on the electron, in terms of e, v_0, and B?

Answer: $F =$ $e v_0 B$

(b) (2) At point A, what is the direction of the force on the electron? (up, down, right, left, into paper, out of paper?)

Answer: direction of force at A is to right

(c) (2) What is the direction of the magnetic field? (up, down, right, left, into paper, out of paper?)

Answer: direction of magnetic field is into paper
[other version: out of paper]

(d) (6) Set the centripetal force, in terms of m, v_0 and R, equal to the magnetic force, in terms of e, v_0, and B. Then obtain the magnitude B of the magnetic field, in terms of m, v_0, R, and e.

$$\frac{m v_0^{\cancel{2}}}{R} = e \cancel{v_0} B \Rightarrow \boxed{B = \frac{m v_0}{e R}}$$

(e) (4) Now suppose that the speed v_0 of the electron is 2.0×10^6 m/s. Calculate the magnitude of the magnetic field B that will cause the electron to follow the semicircular path shown in the figure.

Answer: $B =$ 2.3×10^{-4} T

$$\boxed{B =} \frac{(9.11 \times 10^{-31})(2.0 \times 10^{6})}{(1.60 \times 10^{-19})(0.050)} = \boxed{2.3 \times 10^{-4} \text{ T}}$$

[other version: 0.835 T]

(f) (4) Calculate the time (in seconds) required for the electron to move from A to B.

Answer: time = 7.9×10^{-8} s
or 79 ns

$$\boxed{t =} \frac{2\pi R}{v_0} = \frac{(\pi)(0.05)}{2.0 \times 10^{6}} = \boxed{7.9 \times 10^{-8} \text{ s}}$$

[other version: 3.9×10^{-8} s = 39 ns]

Note: values of e, $m_{electron}$, m_{proton}, and $u = \frac{1}{2}\epsilon_0 E^2$, $U = -\vec{\mu} \cdot \vec{B}$ were added to formula sheets.

2. The plane of a 5 cm × 12 cm rectangular loop of wire is parallel to a 0.15 T magnetic field. The loop carries a current of 4.0 A.

(a) (5) Calculate the magnitude of the magnetic moment of the loop.

Answer: magnitude of magnetic moment = $0.024\ A\cdot m^2$

$A = (0.05)(0.12) = 6.0 \times 10^{-3}\ m^2$

$\boxed{\mu =} I A = (4.0)(6.0 \times 10^{-3}) = \boxed{0.024\ A\cdot m^2}$ (with $\vec{\mu} \perp$ to $\vec{B}$)

[other version: $0.096\ A\cdot m^2$]

(b) (5) Calculate the magnitude of the torque on the loop.

Answer: magnitude of torque = $3.6 \times 10^{-3}\ N\cdot m$

$\vec{\tau} = \vec{\mu} \times \vec{B} \Rightarrow \boxed{\tau =} \mu B \sin 90^\circ = \mu B = (0.024)(0.15) = \boxed{3.6 \times 10^{-3}\ N\cdot m}$

[other version: $2.4 \times 10^{-2}\ N\cdot m$]

(to check units: $F = qvB \Rightarrow 1\ T = 1\ \frac{N}{C\cdot \frac{m}{s}} = 1\ \frac{N}{A\cdot m}$

$\Rightarrow (A\cdot m^2)(T) = (A\cdot m^2)\left(\frac{N}{A\cdot m}\right) = N\cdot m$

but you are allowed to assume SI units are ok!)

(c) (5) Because of the torque, the loop now rotates to a position where its plane is perpendicular to the magnetic field. Calculate the change in its energy U. (U is the usual energy of a magnetic moment in a magnetic field.)

Answer: change in magnetic energy = $-3.6 \times 10^{-3}\ J$

$U_1 = -\vec{\mu}_1 \cdot \vec{B} = 0$ since $\vec{\mu}_1$ is $\perp$ to $\vec{B}$

$U_2 = -\vec{\mu}_2 \cdot \vec{B} = -\mu B = -(0.024)(0.15) = -3.6 \times 10^{-3}\ J$

since $\vec{\mu}_2$ is $/\!/$ to $\vec{B}$

$\therefore \boxed{\Delta U =} U_2 - U_1 = \boxed{-3.6 \times 10^{-3}\ J}$

[other version: $2.4 \times 10^{-2}\ J$]

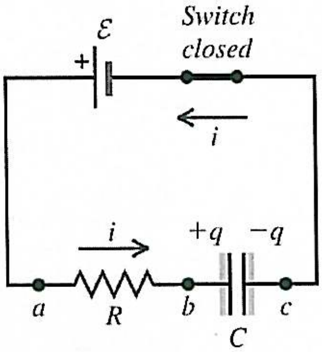
(to check units: $N\cdot m$ just as in part (b), but now interpret $N\cdot m = J$, energy & not torque!).

(d) (5) Where has the lost magnetic energy gone? According to conservation of energy it cannot just disappear.

kinetic energy of rotation (as in motor)

same units but different physical quantity

3. In the drawing, $q = 0$ at time $t = 0$, where q is the charge on the capacitor. Then the switch is closed (at $t = 0$), and the charge begins to build up.



When the switch is closed, the charge on the capacitor increases over time while the current decreases.

(a) (10) Write down the differential equation for the time-dependent charge $q(t)$, in terms of q, $\frac{dq}{dt}$, ε, C, and R.

Kirchhoff loop rule:

$$\mathcal{E} - iR - \frac{q}{C} = 0 \quad \text{with } i = \frac{dq}{dt}$$

$$\Rightarrow \mathcal{E} - \frac{dq}{dt}R - \frac{q}{C} = 0 \quad \text{or} \quad \boxed{R\frac{dq}{dt} + \frac{1}{C}q - \mathcal{E} = 0}$$

(b) (10) You will find the solution for $q(t)$ on the formula sheet for Exam 2. Substitute this solution into the differential equation of part (a). **In a clear, detailed, and well-organized set of steps, show that this solution satisfies both the differential equation and the initial condition** for $q(t)$.

solution given as $q(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$

$$\Rightarrow \frac{dq}{dt} = \cancel{C}\mathcal{E}\left(-e^{-\frac{t}{RC}}\right)\left(-\frac{1}{R\cancel{C}}\right)$$

$$= +\frac{\mathcal{E}}{R}e^{-\frac{t}{RC}}$$

Substitute into equation:

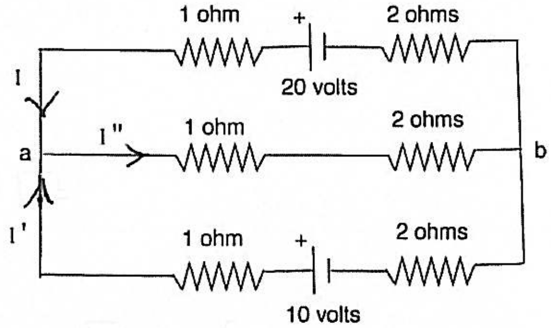
$$\cancel{R}\left(\frac{\mathcal{E}}{\cancel{R}}e^{-\frac{t}{RC}}\right) + \frac{1}{\cancel{C}}\left(\cancel{C}\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)\right) - \mathcal{E}$$

$$= \mathcal{E}e^{-\frac{t}{RC}} + \left(\mathcal{E} - \mathcal{E}e^{-\frac{t}{RC}}\right) - \mathcal{E}$$

$$= 0 \quad \checkmark$$

You can draw arrows for currents as you wish.

4. For the circuit shown in the figure, there are 3 unknown currents, called I, I', and I''.



(a) (10) Write down 3 **independent** equations that can be solved to find these 3 unknown currents. To save time, you can leave off the units. For example, you can write just 20 instead of 20 volts (or 20 V), and you can write just 2 instead of 2 ohms (or 2 Ω).

Kirchhoff's current rule: $I'' = I + I'$ (1) [for this choice of arrows]

loop rule, upper loop: $20 - (1+2)I - (1+2)I'' = 0$ (2)

lower loop: $10 - (1+2)I' - (1+2)I'' = 0$ (3)

[another way is to use big loop & 1 other loop]

(b) (10) Solve these 3 equations to find the values of I, I', and I''.

Answers: $I = \frac{10}{3}$ A, $I' = 0$, $I'' = \frac{10}{3}$ A

One way:

$20 - 3I - 3I'' = 0$ (2) with $I'' = I + I'$ (1)

$10 - 3I' - 3I'' = 0$ (3)

(2) − (3): $10 - 3\underbrace{(I + I')}_{= I''} = 0 \Rightarrow \boxed{I'' = \frac{10}{3}}$

Then (2): $20 - 3I - 3\left(\frac{10}{3}\right) = 0 \Rightarrow 3I = 20 - 10 = 10$

$\Rightarrow \boxed{I = \frac{10}{3}}$

Then (3): $10 - 3I' - 3\left(\frac{10}{3}\right) = 0 \Rightarrow 3I' = 10 - 10 = 0$

$\Rightarrow \boxed{I' = 0}$

[Check: (1) is $\frac{10}{3} = \frac{10}{3} + 0$

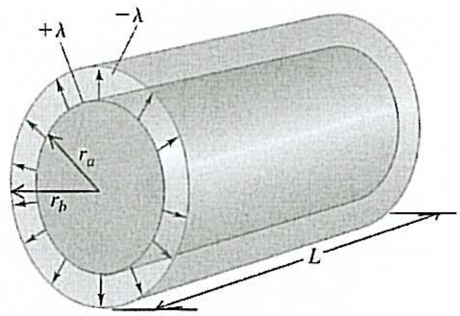
(2) is $20 - 3\left(\frac{10}{3}\right) - 3\left(\frac{10}{3}\right) = 0$

(3) is $10 - 3(0) - 3\left(\frac{10}{3}\right) = 0$]

5. The inner cylinder of a long, cylindrical capacitor has radius r_a and linear charge density $+\lambda$. It is surrounded by a coaxial cylindrical conducting shell with inner radius r_b and linear charge density $-\lambda$. (See the figure.) In doing this problem, you may assume the result

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r} \quad , \quad r_a < r < r_b$$

for the electric field. (This follows from Gauss's law, as we showed.)



(a) (4) Calculate the energy density in the region between the conductors, at a distance r from the axis, in terms of ε_0, r, and λ.

$$\boxed{u =} \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \left(\frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\right)^2 = \boxed{\frac{1}{8\pi^2\epsilon_0}\frac{\lambda^2}{r^2}}$$

(b) (6) Integrate the energy density calculated in part (a) over the volume between the conductors, in a length L of the capacitor, to obtain the total electric-field energy per unit length, U/L.

$$U = \int_{r_a}^{r_b} \underbrace{(L \cdot 2\pi r\, dr)}_{\text{volume in } dr \text{ at } r} \left(\frac{1}{8\pi^2\epsilon_0}\frac{\lambda^2}{r^2}\right)$$

$$= 2\pi \cdot \frac{L}{8\pi^2\epsilon_0} \cdot \lambda^2 \underbrace{\int_{r_a}^{r_b} \frac{\cancel{r}\, dr}{r^{\cancel{2}}}}_{= [\ln r]_{r_a}^{r_b}}$$

$$= \frac{L\lambda^2}{4\pi\epsilon_0} \underbrace{[\ln r_b - \ln r_a]}_{= \ln\left(\frac{r_b}{r_a}\right)}$$

$$\Rightarrow \boxed{\frac{U}{L} = \frac{\lambda^2}{4\pi\epsilon_0} \ln\left(\frac{r_b}{r_a}\right)}$$

(c) (6) Using the fact that $V_b - V_a = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$, calculate this potential difference and then the capacitance per unit length C/L of this cylindrical capacitor, in terms of r_a, r_b, and ε_0.

$$\boxed{V_b - V_a} = -\int_{r_a}^{r_b} \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}\, dr$$

$$= -\frac{\lambda}{2\pi\epsilon_0} \left[\ln r\right]_{r_a}^{r_b}$$

$$= \boxed{-\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_b}{r_a}\right)}$$

Then $C \equiv \dfrac{Q}{|V_b - V_a|} = \dfrac{\cancel{\lambda} L}{\dfrac{\cancel{\lambda}}{2\pi\epsilon_0} \ln\left(\dfrac{r_b}{r_a}\right)}$

$$\Rightarrow \boxed{\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln\left(\frac{r_b}{r_a}\right)}}$$

(d) (4) We derived the expression $U = \frac{1}{2}\frac{Q^2}{C}$ for the potential energy stored in a capacitor with capacitance C holding a charge Q. Using this expression and the result of part (c) for C, calculate U/L again. Does your result agree with that obtained in part (b)?

$$U = \frac{1}{2} \frac{Q^2}{C}$$

$$= \frac{1}{2} \frac{(\lambda L)^2}{\frac{2\pi\epsilon_0}{\ln\left(\frac{r_b}{r_a}\right)} \cdot L}$$

$$\Rightarrow \boxed{\frac{U}{L} = \frac{\lambda^2}{4\pi\epsilon_0} \ln\left(\frac{r_b}{r_a}\right)}$$

which is the same as in part (b)

6. (5 points extra credit) Describe 5 **distinct** applications of capacitors. (We mentioned 8 in class.)

(i) energy storage for electronic flashes on cameras

(ii) various roles in electronic circuits

(iii) tuners in radio and TV receivers

(iv) touchscreens in, e.g., iPhone

(v) sensors for automobile airbags

etc. [automobile turn signals, heart pacemakers, high energy pulsed lasers, ...]

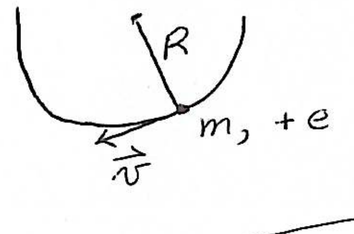
7. (5 points extra credit) You are given a sample of natural gas, and are asked to determine its composition. That is, you are supposed to perform an experiment to learn what percentage you have of each kind of molecule. You can singly-ionize each molecule, so that you have CH_4^+, $C_2H_6^+$, H_2^+, etc.. You can then subject these ionized molecules to an electric field and a magnetic field.

How can you use these two fields to determine the mass of each molecule (and thus its chemical identity)?

In other words, explain clearly and in detail how a mass spectrometer works, using a drawing and the relevant equations.

Accelerate ion to velocity $\vec{v}$ with electric field $\vec{E}$.

Then measure radius R of its path in magnetic field $\vec{B}$.



$$\frac{m v^{\cancel{2}}}{R} = e\cancel{v}B \Rightarrow \boxed{m = \frac{eBR}{v}}$$

[We can also use a velocity selector with $eE = evB \Rightarrow v = \frac{E}{B}$.]